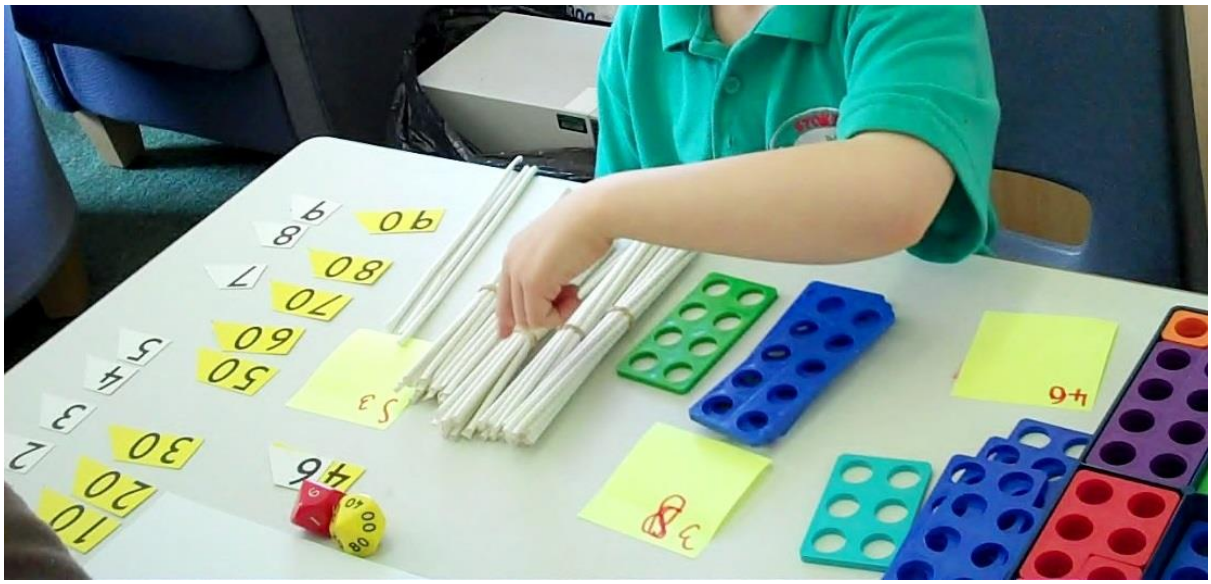
A Practical Guide to Supporting Children who Struggle with Mathematics



Babcock LDP Primary Maths Advisers
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1. Introduction

The purpose of this guidance is to provide schools with a range of practical interventions and approaches in order to meet the needs of children and young people with severe and persistent mathematical difficulties. This guidance can also be used in conjunction with A Devon Approach to Understanding Children and Young People with Mathematical Difficulties and A Devon Approach to Supporting Children and Young People with Mathematical Difficulties which were written in collaboration with the Babcock LDP Educational Psychology Service (2016).

When supporting children who struggle with maths, it is crucial to identify their current secure skills, knowledge and understanding as well as any gaps and misconceptions so that the support provided is tailored to the needs of each child.

This is best done through one to one diagnostic assessments where the teacher can explore a child's understandings to find out what they know, what they can do and how they do it. It is also important to establish with which aspects of maths they are confident, what helps them to learn and with which equipment they are familiar. During these assessments, there should be opportunities to listen to how the child explains and uses mathematical language and observe how they represent their thinking.

Many children who struggle with mathematics will lack confidence and may be reluctant to even attempt to solve problems. Therefore, these diagnostic sessions should provide immediate success for the child, helping them to recognise what they know and to understand that support will enable them to build on this knowledge. Developing positive relationships and expectations in a safe learning environment is imperative if a child is to feel confident to take risks with learning.

Kay and Yeo (2003) identify that when using mathematical images they must be transparent to the child and if not, an alternative should be used. Therefore it may take several attempts to find the image that suits each child for each mathematical concept. A good image for teacher may not be the best image for the child. E.g. the teacher struggles with seeing the benefit of using bead-strings and finds them fiddly, but the child enjoys the sensation of moving beads which supports their conceptual understanding of more and less.

Some familiar images may in fact be unhelpful for dyslexic or dyscalculic children for example: hundred squares (DfES 2001). Children need to understand how to interpret the structure of the square and make sense of the associated language anomaly: the numbers that are lower in the square are higher numbers.

Developing conceptual understanding through the use of the connective model to plan and teach mathematics is important for all children. It is essential that children should understand mathematics; that they should make sense of what they are doing and not just learn to reproduce learnt procedures (Haylock 2010).

Children with mathematical difficulties who are identified as having memory difficulties will struggle to remember procedures and therefore are most vulnerable to not making good progress if teaching does not focus on conceptual understanding. Ofsted describe outstanding teaching as that which 'is rooted in the development of all pupils' conceptual understanding of important concepts and progression within the lesson and over time (Ofsted 2013).

Therefore children who are identified as having specific mathematical difficulties would benefit from quality first teaching that is based on developing children's conceptual understanding and this can be achieved by planning maths activities around the connective model. In addition they will also benefit from individual or small group support to develop

their reasoning where they learn to derive facts from their *own* known facts – where the teacher spends time finding out what the child already knows and then plans how to support them to derive further facts, planning activities that connect context, symbols images and language.

This document provides a bank of ideas to support pupils who struggle with mathematics. Some pupils may have specific learning difficulties such as poor working memory or an inability to retain or recall key facts. Some pupils may respond well to exploring a range of strategies for solving a problem, whereas for others having one strategy works better. Knowing the specific issues and needs of each child is crucial. The following ideas are suggestions which can be easily adapted to meet differing needs.

2. The Connective Model

Understanding mathematics is made easier by identifying and understanding connections between mathematical ideas. Teaching and learning of mathematics should therefore focus on making connections. The connective model helps to make explicit the connections between different mathematical representations: symbols, mathematically structured images, language and contexts. This is set out in the model below, adapted from Haylock and Cockburn (1997):

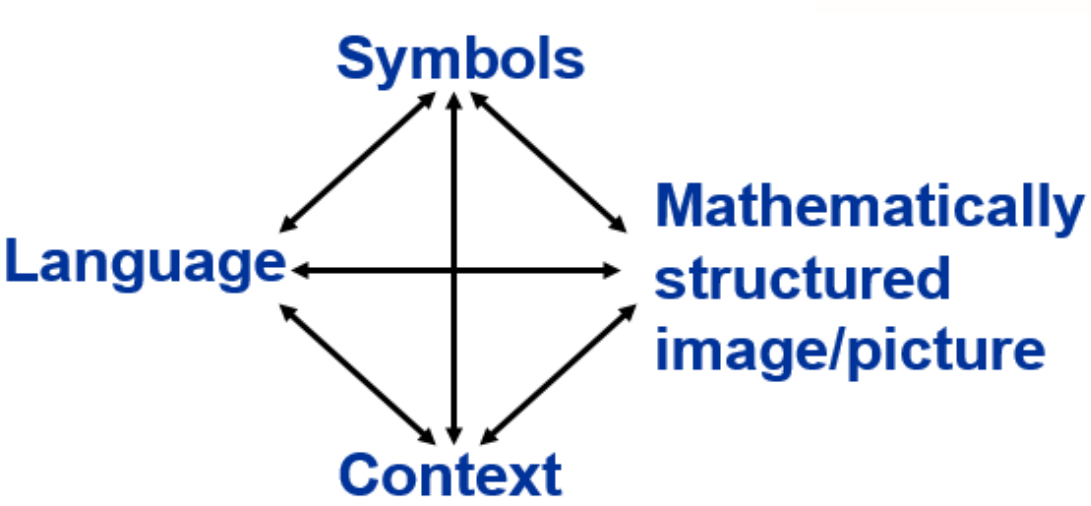


Figure1.1 The connective model of learning mathematics (adapted by the Babcock LDP Primary Mathematics Team from Haylock and Cockburn 1989)

Haylock and Cockburn suggest that it is in making connections between their experiences of these different elements that children's learning is more deeply embedded and their understanding broader and deeper.

For example:

- A class might be looking at how to reduce their school's impact on the environment and decide to focus on how the pupils travel to school. This provides the context and purpose for the mathematics.

- The pupils examine the data that has been collected, which shows the number of pupils who walk, cycle or travel by car, bus or taxi, and talk about what they notice and what they would like to change; this will involve both mathematical and contextual language, being used a part of purposeful talk.

- A question arises about the total number of pupils who travel in a vehicle that uses fuel compared to the number of pupils who travel in a sustainable way. This provides

an opportunity for the pupils to look at the most efficient way to add the numbers, using what they know and understand.

- A mathematical image might be used to support understanding of a method; for example, understanding that for 165 + 25 it is easy to partition the second number and use the known facts 5 + 5 = 10 and 7 + 2 = 9 (for 70 + 20 = 90). The image might also be used by a pupil to support their explanation of their method, allowing others to access their thinking.

- Images for addition include base 10, number lines, bead strings and place-value counters and the focus should be on using the image to support understanding, not 'to do' the maths. Counting in ones should be avoided and decisions about which images to have available should depend on them being fit for purpose and decisions about which to use to support explanations should be made by the pupils. The symbols might record the steps taken or the calculation undertaken (165 + 5 + 20 = 190 or 165 + 25 = 190) and the pupils should be able to explain the symbols in relation to the maths of the context, which will involve revisiting relevant language.

It is important to understand the role of classroom talk in relation to this model. It would be easy to assume that talk sits within the *language* aspect of the model. But this is the very specific mathematical language and vocabulary used in connection with the experiences the children are having. The role of talk is to help the children make ***the connections themselves***. This talk can take the form of teacher questioning demanding children make connections, children questioning concerning connections not seen, talk between children, and explanation of points of view etc. The verbal accompaniment to the children's experiences is what allows them to frame their understanding.

There is also now brain evidence that connecting up these different representations of mathematics leads to more powerful learning. When students work with symbols, such as numbers, they are using a different area of the brain than when they work with visual and spatial information, such as an array of dots. Park and Brannon (2013), found that mathematics learning and performance was optimized when the two areas of the brain were communicating. Additionally, they found that training students through visual representations improved students' maths performance significantly, even on numerical maths, and that the visual training helped students' understanding more than numerical training.

References:

Haylock, D. and Cockburn, A. (1989), *Understanding Early Years Mathematics* pp 2-4.
Park, J., & Brannon, E. (2013). Training the approximate number system improves math proficiency. *Association for Psychological Science*, 1–7.

3. Quality first teaching and learning

Teaching mathematics should include:

- Opportunities for mathematics in both the inside and outside environment, this would include the construction of a number rich, or "numerate" environment;
- Awareness of mathematics in the wider world;
- Direct teaching that is strategic and based on assessment for learning;
- The use of flexible grouping, play based activity and guided work;
- A level of challenge that will allow a child to experience struggle;
- Links across and within the curriculum;
- Assessment that is based on observation and discussion - assessment should be used as the basis for quality teaching.

Learning mathematics should be:

- Experienced within meaningful and purposeful contexts, these could include stories, songs, games, role-play and rhymes; also "real world" situations that demand calculation for the solution of problems.

Learning and teaching in Numbers as Labels for Counting should focus on:

- Progression in counting skills including:
    - Attending to both cardinal and ordinal aspects of number;
    - Exposure to larger numbers and opportunities to explore them;
    - The different experiences of counting e.g. out loud, counting things you can touch, counting things you can't touch, counting events, counting on a line, subitising…etc.
    - Understanding the link between calculating and counting.

Children should be encouraged to independently apply counting skills in a range of different situations. Adults should observe with a view to taking opportunities to mathematise children's independent activity

Learning and teaching in Calculating should focus on:

- Development of calculation skills:
    - Understanding one more/ less as an overlap between counting and calculation "*counting is the first act of calculation…*":
    - Developing understanding of the cardinality and ordinality of numbers "*a sense of number…*" this includes subitising:
    - Laying the foundations for addition/ subtraction and multiplication/ division:
    - Awareness that new knowledge, strategies and number facts can be derived from existing knowledge.

Children should experience situations that require calculation for the solution of problems.

Learning and teaching in Geometry and Measures should:

- Focus on interacting with the properties of shapes and not just on naming them.
- Include many different opportunities to experience positional language – these opportunities will be found across the curriculum.
- Involve the identification, discussion and reproduction of pattern.
- Make connections between geometry, measures, numbers and counting.

Children should encounter measures in contexts where there is a clear need, including standard and non-standard units.

4. Supporting children to develop Number Sense

Number sense is about understanding our number system, starting with counting numbers, building an understanding of how our numbers work and fit together and includes exploring place value and comparing and ordering numbers, applying this understanding in different contexts.

What teaching strategies promote early number sense?

A number of strategies that help to support developing number sense are described by Tsao and Lin (2012). They suggest that children need opportunities to:

1. Work with concrete materials and familiar ideas
2. Compose and recompose different arrangements and representations of number
3. Discuss and share their discoveries and solutions
4. Investigate the realistic uses of number in their everyday world
5. Explore number patterns and relationships
6. Create alternative methods of calculation and estimation
7. Solve realistic problems using a variety of approaches
8. Calculate for a purpose rather than just for the sake of calculating
9. Gather, organise, display and interpret quantitative data
10. Measure and estimate measure for a purpose
11. Explore very large numbers and their representations including using number lines

Ideas for developing each of these can be found here http://nrich.maths.org/10737

5. Ideas and activities for overcoming common errors and misconceptions in number

5.1 Oral Counting

Oral counting is the ability to correctly order a string of numbers. Initially this will involve memorising this string, therefore early experiences focus on learning these sequences both forwards and backwards with reference to the five stages outlined by Fuson (1988)

String Level - a continuous sound string
Unbreakable List level – separate words but the sequence can't be broken and always starts from 1
Breakable Chain Level – start to count from any point
Numerable Chain Level – sequence, count and cardinality are merged so, if you are counting from 3, then 3 is the first number, 4 is the second number …
Bi-directional Chain Level – numbers can be said in either direction and start at any point

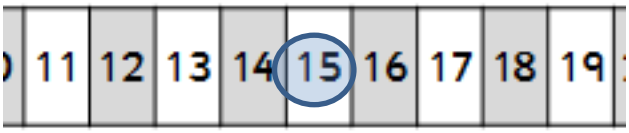
- *A common error is mispronouncing and confusing the 'teen' and 'ty' numbers, influencing the reading and writing of numbers and the understanding of place value*
- *Be aware of children who count by rote when counting in steps. Ensure concrete resources and contexts are used to develop a clear understanding of what these steps mean*
- *Many children can remember a short sequence when counting in steps but struggle to explain the pattern and/or use this to work out the next number*
- *Look out for children counting forwards in tens who may say: 70, 80, 90, 20. When counting backwards many recite 40, 30, 12, 10. This relates to confusions between 12 and 20 and between 'teen' and 'ty' numbers.*
- *Many children might appear fluent when counting forwards but struggle to count backwards. Counting backwards is no more difficult than counting forwards; they just have not practised this sequence enough.*
- *If counting backwards is particularly difficult, encourage additional practice in oral counting into higher value sequences. Provide support by presenting sequences with missing numbers. Focusing on pattern can be particularly helpful for children with memory problems. Ensure appropriate visual support is provided for counting through key boundaries and for counting in tens to ones.*

✓ Play games that involve counting/chanting. Use different voices, speeds and volumes to recite numbers e.g. high pitched; animal voice; playground voice; sing-song voice. The child chants numbers, whispering or shouting the numbers in a pattern.

✓ Talk about numbers that are special to the child e.g. I am 6; I live at number 15; the shirt number of my favourite football player is 11. Find these numbers on a number line and orally count on or back using the number line to help.

✓ Build up their repertoire of songs and rhymes to encourage rhythmic pattern-making and counting e.g. Ten green bottles; One, Two, Three, Four, Five, once I caught a fish alive; There were ten in the bed; Five currant buns in the baker's shop etc.

- ✓ Develop a range of activities to select a starting number from which to count forwards and backwards e.g. pick a number card from a feely bag, from the puppet's pouch or hidden in the sand or ask the child to select a number with a particular property suitable to their experience e.g. a number less than 20; a 2-digit number; an odd number; a multiple of ten; a number between 30 and 40 etc.



- ✓ Use a puppet and ask the child to help it with some counting e.g. count to 20 and back with the puppet using different voices, the puppet whispers the counts, stops and the child shouts the next three numbers, change roles. The child corrects mistakes the puppet makes e.g. number/s omitted; numbers in the wrong order; numbers repeated; errors such as saying: thirteen, fourteen, fiveteen … nineteen, tenteen/twenteen … twenty-nine, twenty-ten …

- ✓ Start the count – one child rolls a dice to start the count. Take it in turns to say the next number until one person says 20. This person gets a counter – First to get 5 counters. Adapt by changing numbers on dice or counting on or back in different steps.

- ✓ Count on ones, twos, fives or tens when doing rhythmic activities e.g. swings, jumps, throwing a ball or bean bag.

- ✓ Use a range of objects for the child to count in twos, fives or tens. Try to make the images relate to counting in real life situations e.g. pairs of socks, gloves/handprints, coins.

- ✓ Use dot pattern cards and arrays to show how numbers can be put together.

- ✓ Use a calculator that has been set up to add one, two, five or ten each time the equal key is pressed. Ask the child to predict the next number in the sequence prior to revealing it on the calculator display. Ask questions such as: *'If we keep going will we see 24?' 'Stop at the number nearest to 21, 34 …'*

5.2 Counting Objects

Gelman and Gallistel (1978) maintain that children's understanding of what they are doing when counting is grounded in five principles:

How to Count	The One-to-One Principle	Assigning a distinct counting word to each item, even if a child says '1, 6, 2'.
	The Stable Order Principle	Knowing that the list of words must be a consistent one, even if a child repeatedly counts '1, 3, 2, 6'. We should not be surprised if children make up their own lists of numbers, given the inconsistencies in our counting system.
	The Cardinal Principle	Counting leads to a 'product' at the end. If a child has to count again in response to 'How many …?' they have not grasped the principle.
What to Count	The Abstract Principle	Counting collections of abstract, miscellaneous items, even if a child refers to them as 'things'.
	The Order Irrelevance Principle	Knowing that the order of what is being counted is irrelevant – when a child has grasped this, they know what they are doing when counting.

Researchers have developed these ideas, including Trundley (2007) who identified The Five Themes of Counting:

Matching number names to 'items' in a one-to-one correspondence	The culture of counting to which the children are being introduced is one that has movement at its centre…moving is the consistent part of the counting process as children first experience it and…is what they attend to first. …This can lead children to first consider correct counting to involve one-to-one correspondence of a word and a movement. **Challenge: matching a count to the objects creating a set**
Organising a count	Organising a count …focuses on children taking responsibility for the count themselves and making decisions about how they will find out 'How many?' including keeping track and using what they know about numbers, through subitizing and partitioning. This moves counting beyond being a purely skill-based, procedural

	activity to one that includes application of mathematical thinking and is the first step towards developing deductive strategies. **Challenge: using what you know and keeping track**
Conservation of number	Understanding what it means to say there is a certain number in a counted set is at the core of conservation of number. Conservation of number underpins an understanding of equality and is one of the earliest experiences of "Same value, different appearance", one of the foundations of algebra. **Challenge: labelling the set rather than the objects**
The language of numbers	The speaking, reading and writing of numbers develop alongside each other. Whilst memorising spoken numbers is the first part of the language the children learn, once they start to engage with the symbolic representation, their learning can be triggered by engagement with any of the language elements. This then feeds into and connects with the other two. **Challenge: allow access to numbers beyond those that can be read or written to expose the pattern of the counting numbers**
Understand why we need to count and what happens to numbers whilst counting	This is all about purpose and understanding; attending to the additive structure of the number system and how counting numbers fit into the number system. Understanding counting is an essential support to the other four themes and is the mathematics of counting; it includes making connections with addition and subtraction

Trundley identifies understanding of cardinality as a key aspect of early understanding of number and expands on the Gellman and Gallistel definition:

> *An understanding of cardinality is built on an ability to subitize and attach number names to small numbers and includes recognition of the fact that these small numbers can be partitioned in a variety of ways including previous number +1, leading to an understanding of the relationship between successive numbers, as well as being able to say the number name for the whole set.*
>
> Trundley (2007)

Some children do not grasp the cardinal property of number, even though they might apply the number-name sequence to objects and actions. This is apparent when children do not recognise that the final number word represents the numerosity of the set, appreciate that the order objects are counted is irrelevant or are not assigning one counting word to each object being counted. They may also not understand the relationship between two consecutive counting numbers.

- *Discuss strategies for keeping track of which objects have been counted.*
- *If static objects or drawings on a page are being counted, encourage the child to mark each item as it is counted.*
- *If moveable objects are being counted, ensure the child points to each object as they say the numbers or moves them away from the group, rather than just scanning with their eyes.*
- *Encourage children to organise the objects as they count them – lining them up helps them make connections with a number track or line but there may be better ways of organising small sets so that the number can be seen without counting.*
- *Ensure children have experience of counting sets of given objects and also of counting out a given number from a larger set.*
- *Once children can count in steps of two or five for example, encourage them to organise their count by grouping in sets and then counting.*
- *When counting backwards, children need to relate this to a group of objects getting smaller/less and focus on the group that is left, not the objects being removed.*
- *When counting, provide clear instructions on how to count in an organised, meaningful way. Make sure numbers have meaning for a child, so provide a relevant context.*

✓ Use simple percussion instruments (drum, tambourine, wooden blocks, triangle) to model a number of beats. Secretly record the chosen number. The child listens and identifies the number of beats and reveals the hidden number to check. Ask the children to count the beats silently but shout at every tenth beat.

✓ Say a number each time you throw or bounce a ball, jump over a skipping rope or clap your hands.

✓ Use a 'thing on a string' for the child to count at each swing. The child could whisper on odd numbers and shout even numbers. Set up different challenges using the swings e.g. pick a 2-digit number and count backwards, crossing the tens boundaries.

✓ Put numbered counters into a bag. Draw a counter from the bag and place it in position on an empty number track by counting from zero. Draw other counters from the bag and position in the same way. To extend learning, encourage the child to count on or back from numbers already placed on the track.

✓ Drop objects into a tin. The child listens and says how many and then checks by counting. Encourage the child to place the objects in a line. Ask questions such as: *'Are there the same number of objects if we spread them out? … place them in a circle? … Move some over here?'*

✓ Have different posting boxes with numbers written on them for the child to post that number of envelopes into the box. Relate this to real life e.g. school Christmas post, collections of school vouchers. *'How many altogether? How many for each class? How many groups of ten?'*

- ✓ Use grouping strategies to count larger numbers of objects e.g. pairs of socks; piles of pennies; towers of bricks; bags of sweets.

- ✓ Dice board games such as Snakes and Ladders or Ludo or any number track game which involves rolling a dotty dice and moving that number of spaces. Make sure the children count the number of jumps and don't say 'one' until they have jumped forward one space.

- ✓ Dice can be used with any collection of objects. E.g. take turns to roll the dice and build a tower of bricks. First to ten/twenty. Children have a collection of 10 different small items of their choice. Take turns to roll the dice and take that number from your opponent. Winner will be the player with most or all items after a set number of rolls or when time is up. Rules can be adapted to meet the needs of the group.

5.3 Reading and Writing Numbers

- *A common error is the confusion of the numeral '3' and '5'. This might occur due to issues with miss-hearing and mispronouncing the word 'three' for 'free'. It is important to emphasise the pronunciation and connections between 'three', 'thirteen' and 'thirty' and 'five', 'fifteen' and 'fifty' when appropriate.*
- *Other numbers that are often confused are '12' and '20'*
- *All 'teen' and 'ty' numbers can be confused as the identifying sounds occur at the end of the number words. "Teen" numbers can be very tricky if the children do not realise that "teen" means 10, and the usual rule of saying tens before ones is broken. For example, sixteen means literally "six and ten", which really should be "ten and six". Also 11 and 12 break the "teen" pattern by not being "oneteen" and "twoteen". Further confusion can occur for the children who fail to realise words ending in "ty" are tens. Many children confuse "sixty" with "sixteen". Persistence in teaching "teen" and "ty" is needed to overcome these problems*
- *It is important not to get stuck in the 'teen' numbers as moving on to work with larger numbers reveals the structures and patterns in our number system.*
- *Many children confuse 'teen' and 'ty' numbers and may appear to not remember how to say or write 70 and 17 but with support to gain conceptual understanding of the number system using the connective model they may learn to reason 'what is the same' and 'what is different' about seventeen and seventy.*

✓ Feely bag containing plastic or wooden numbers. Play 'guess the number': the children have to say and write. Using two numbers in the bag, ask the child which one and two digit numbers they can write e.g. 1, 7, 17, 71

✓ Practise writing numbers with correct formation as part of handwriting. Do this in a variety of ways such as 'rainbow numbers' (tracing over a large scale written number with lots of different coloured pens), water and large paintbrush on the playground or outside walls, glitter trays, sand …

✓ Read number books and rhymes. Children can make their own zig-zag number books.

✓ Read these numbers on the number track. Write the missing numbers in the boxes.

6 7 8 ❒ ❒ ❒ 12 13 14

✓ Use a variety of 'real life' resources that display numbers e.g. telephones, remote controls, clocks, cash registers, calculators, key pads

- ✓ Make a magnetic fishing game together. Ask the child to write a different number on each fish and decorate it. Ask questions such as: *'Can you catch fish number …?' 'What number have you caught?'* Also do this with number words.

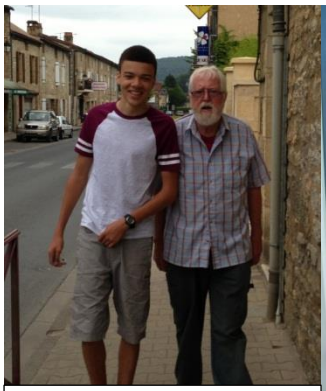
- ✓ Dot to dot pictures

- ✓ Role play games such as 'Farmer'. Write a list of numbers of animals to buy at the market e.g. 6 cows, 10 sheep, 7 pigs … Go to market, read the list and buy.

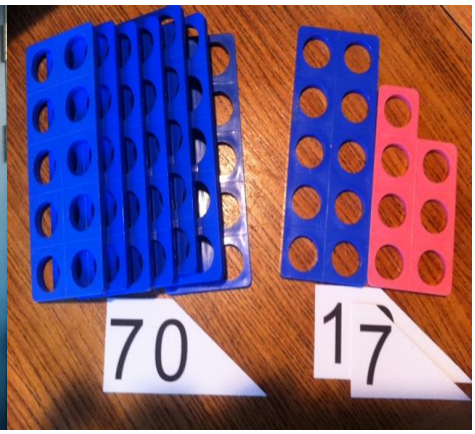
- ✓ Bingo games

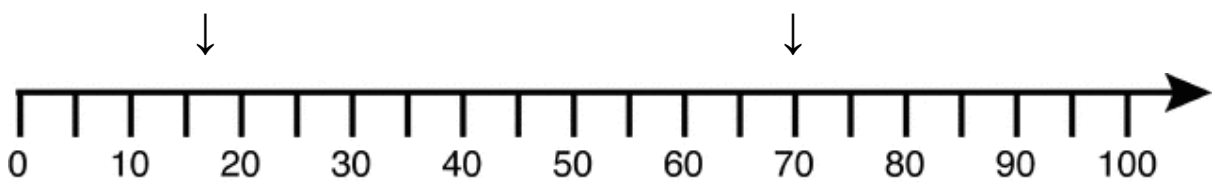
- ✓ Use words, numbers and images to support understanding of 'teen' and 'ty' numbers such as: cups of tea and coffee/magic beanstalk or 'teen'agers. Write the numbers on the chosen image and place on display such as 100 square or number line.

- ✓ Sort 'teen' and 'ty' numbers to match correct number of coins or other sets of objects along with structured maths resources such as Numicon, place value arrow cards, bead strings and base 10.



Isaac is 17 and Michael is 70





5.4 Comparing and Ordering Numbers

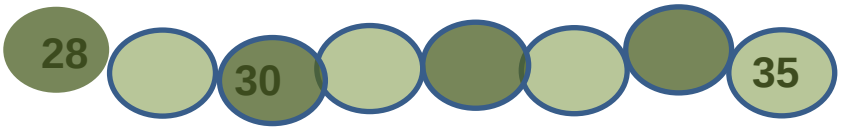
The language of position in number sequences can be confusing … before, after, next, between, in front, behind, more, less, fewer, greater etc. The meaning of these words might change depending on whether we are counting forwards or backwards i.e. when counting backwards the number before 13 is 14!
The visual prompts used to support this thinking are crucial. It is important to consider a child's view of an image e.g. when looking at a 100 square which has 1 in the top left hand corner and 100 in the bottom right corner, it can be confusing to describe numbers as higher or lower for example when their position does not match this description.

Language confusion may also be evident when comparing numbers. For example, when asked to identify a bigger number some children will interpret this as the size it is when drawn not the relative value.

- *Children are usually familiar with the vocabulary relating to 'more' but often are not so familiar with language and experiences relating to 'less'.*
- *Opportunities should be given for children to compare sets of objects and numbers using a range of vocabulary. They should be able to say how many more or fewer one group is than another. Children may misinterpret the word 'more' for 'how many altogether?', yet when asked 'how many extra are there?' may be able to answer correctly.*

- ✓ Create an attractive number line with the child based on a theme of their interest. Use this number line to order numbers from smallest to largest, largest to smallest, reorder 'naughty numbers' which have mysteriously moved places, find missing numbers. Present the number line horizontally and vertically at various times.

- ✓ Ask questions such as: *Which numbers come between 30 and 35? What number comes between 28 and 30? After 35? Before 28?*



- ✓ Set up a shop with items labelled with different prices. Ask the child to sort items into baskets marked with a price range e.g. 4p to 9p; 10p to 15p; 16p to 20p; 21p to 25p. Ask questions such as: '*If you have 20p to spend, which baskets can you choose from? Why?'*

- ✓ Chicken Pox faces (spotty faces with number of spots on pyjama pocket). Compare two numbers and say which is more or less and give a number in between.

- ✓ What's my number? The child asks questions to identify a mystery number by elimination. Encourage use of language of comparing and ordering by providing prompt cards such as: '*Is it bigger than …?' 'Is it smaller than …?*'

- ✓ Rearrange a hundred square into a number track and vice versa.

- ✓ Provide several containers holding different numbers of objects. Ask questions such as: '*Which pot has the most? The fewest? The greatest number? Order the pots from the smallest number to the greatest number. Which container holds a number closest to 10? 20?'*

- ✓ In pairs, take turns to throw one or two dice (1-6, 1-10, 1-20 …). Each child builds a tower to match their throw with linking cubes to match the number shown. Stand the towers next to each other to find the difference.

- ✓ In pairs, play 'Keep the Difference' – roll a die and say your number. Take this many counters and put them in a line. Ensure the lines are under each other. Work out the difference. The player with the most counters wins the difference. Relate this to the number line/track.

5.5 Place Value

Our number system is based on the principle that all numbers can be represented by using just ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). The value of each digit is determined by its place in a number. Pupils need to understand the additive structure of our number system, for example: 364 = 300 + 60 + 4 as well as the multiplicative structure, with each place being ten times the size of the place on its right.

- *Children who have not yet understood place value may say that eight hundred and ninety five is bigger than one thousand.*
- *Numbers that include zeros often pose particular problems. For example, a child might write one thousand and six as 10006. Careful teaching, using a range of structured mathematical resources is crucial. Place value charts or mats might be useful to organise resources and provide a clear image. Manipulating different representations of the same numbers supports an understanding of place value.*
- *Structured resources such as base 10, Numicon, bundling straws, place value arrow cards and bead strings provide clear images for understanding place value.*

✓ Ask the child to count out a given number of linking cubes between 11 and 19. *E.g. 16.* Ask them to connect 10 of the cubes together to make a 10 rod. Place cubes on a place value mat. Say *'One ten and six ones is sixteen.'* Write 16 underneath on the place value mat. Ask the child to find the matching place value arrow cards.

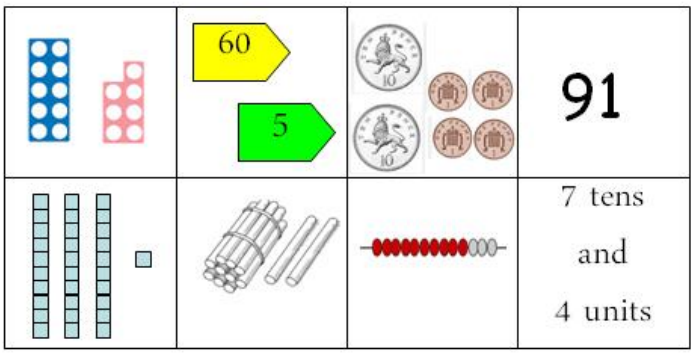
✓ Take two digit cards e.g. 7 and 2. Ask: '*Which two digit numbers can you make?'* (27 and 72) Use single straws and bundles of ten straws to represent each number. Ask questions such as: '*What is the value of the 7 in 27/72?' 'How many tens/ones in this number?' 'Which number has more/fewer tens/ones?'* It is important that children recognise that there are 72 ones in 72. Also include examples with zero.

✓ Role play using real coins (1p and 10p coins) e.g. paying for items at a toy shop, cake stall or games at Christmas Fayre.
Model recording as a number sentence e.g. 34p = 30p + 4p and then encourage children to write price labels and find matching coins.

- ✓ Use a combination of place value arrow cards and concrete materials e.g. bundling straws, Numicon and bead string to illustrate place value.

- ✓ Treasure chest game. Roll a dice, collect 1p coins and every time you have collected 10, convert them to a 10p coin. Keep a running total. Who has the most treasure?

- ✓ Place value bingo (use a range of structured resources). Match numbers to images.



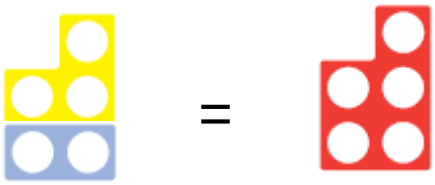
- ✓ Make a two digit number – use two sets of 0 – 9 digit cards or dice. Each player takes two cards or rolls two dice and makes the highest/lowest two digit number.

5.6 Beginning to calculate - using known facts, including understanding of inverse relationships

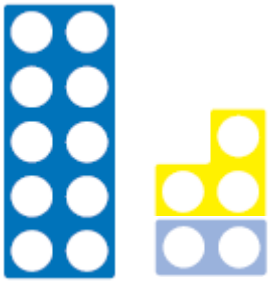
Calculating mentally involves working memory when solving a problem. First you need to remember each number and each stage of the calculation, then recall and use any relevant known facts, retrieve and use an efficient strategy and then give an answer. If working memory, or remembering/ recalling facts are an issue, carefully scaffolded support needs to be put in place so children are able to work on solving a problem rather than struggling to recall what it is that they know that might be useful. This might mean providing visual prompts for strategies and number facts previously explored e.g. relevant patterns of number bonds or multiplication facts.

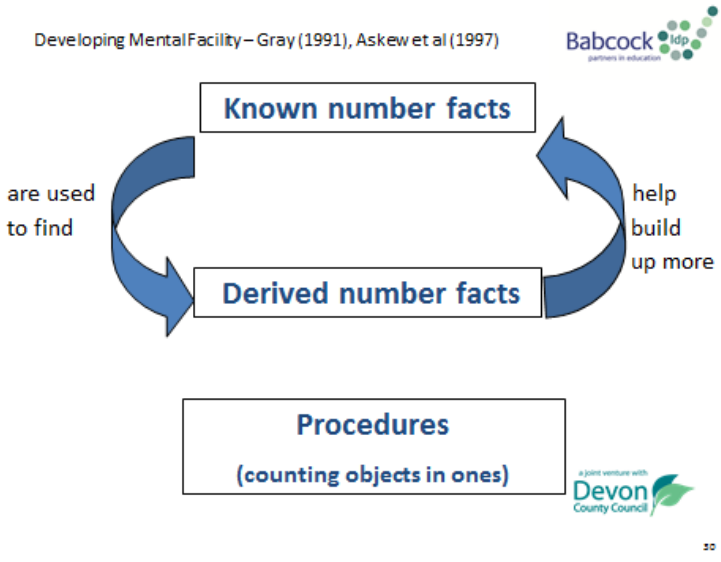
It is important to have a clear understanding of the child's known facts and to remind them which are relevant and useful when solving problems. For example, it might be helpful to say: 'I know you know 5 add 5, how will this help with 5 add 6?' or 'If we know that one less than 8 is 7, how does that help with one less than 28?' All of these questions encourage children to make connections and generalise their understanding. Ensuring all elements of the connective model are present will support children in understanding how and when to use what they know.

For example, having established that 3 add 2 is 5



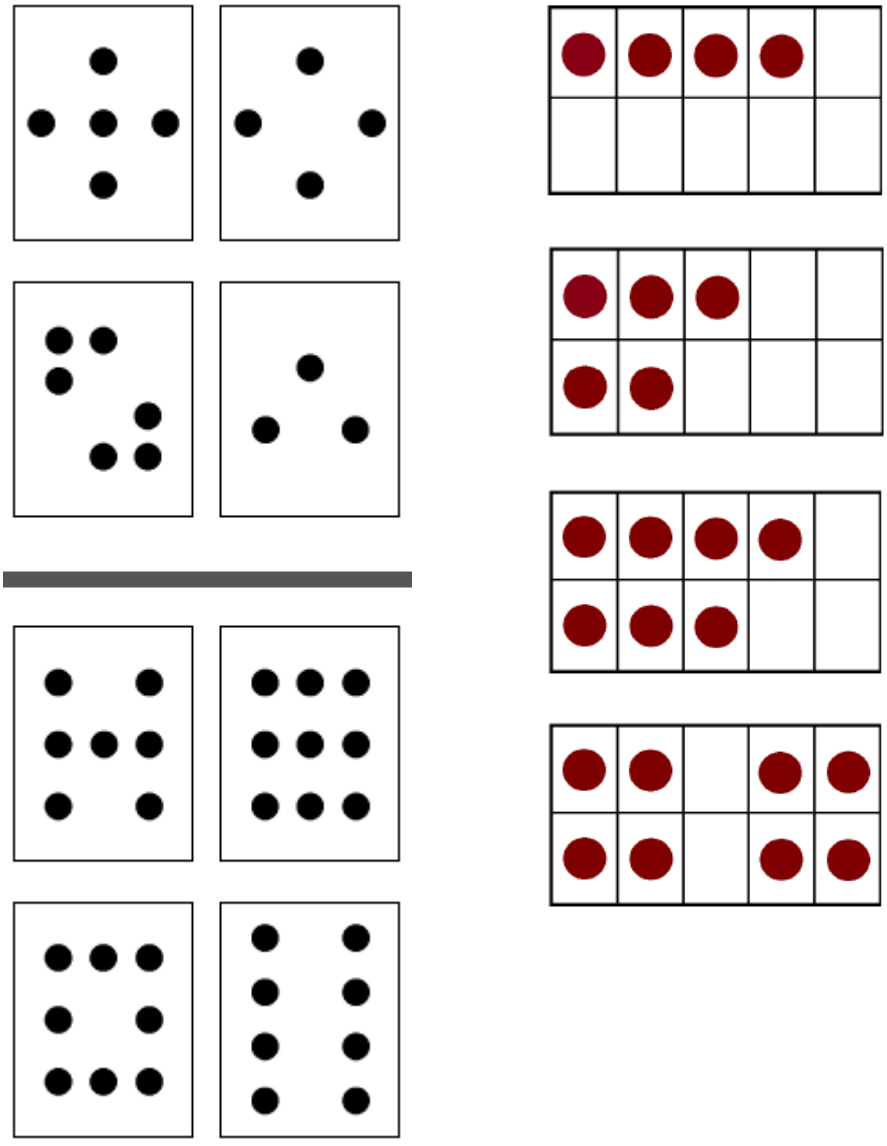
This fact can then be used to work out 13 add 2 and further patterns can be explored.





The first step in moving on from counting everything is using what you know. This includes subitising, understanding the cardinality of numbers and an ability to 'count on' from a given number. Children often need prompting to use what they know. This includes recognising a small set and counting on to find 'how many?' Being able to subitise (recognise how many without counting) and recognise iconic representations (familiar patterns for numbers such as dots on dice) are key skills.

Resources such as dot pattern cards and ten frames can be used to encourage children to look for patterns they know and count on or use known facts to calculate 'how many?'



- ✓ Use a variety of practical resources to represent a calculation *e.g. 'There are 3 ladybirds on one leaf and 2 ladybirds on another leaf, how many ladybirds altogether?'* Encourage the child to use the correct mathematical vocabulary, leading to writing a matching number sentence. Ensuring links to subtraction are made alongside addition is crucial to children understanding and using inverse relationships. '*If 3 ladybirds fly away, how many will be left?'*

- ✓ Give a selection of symbols and numbers appropriate to the child. Can they make a number sentence, represent it using resources and explain it using appropriate language?

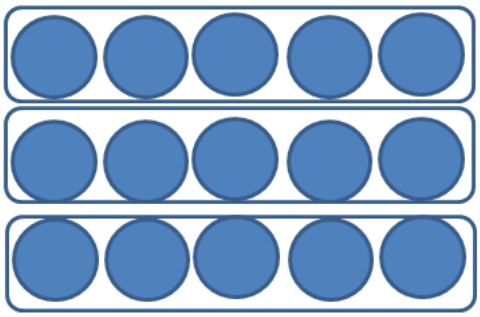
- ✓ Ask questions with a hidden amount such as: '*I have ten pennies in my two pockets. In this pocket I have 4p. How much money is in my other pocket?' 'The pet shop has 8 fish in two tanks. There are 5 fish in this tank. How many will there be in the other tank? How do you know?'*

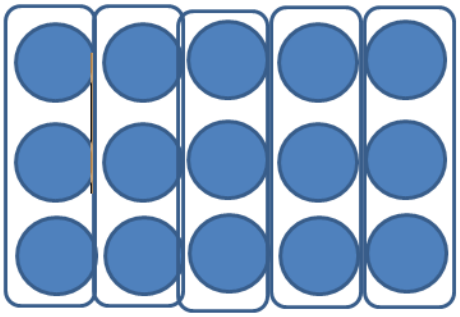
- ✓ Use a pan balance to demonstrate different calculations. Include all four operations and missing number sentences.

- ✓ Generate two or three digit numbers using place value dice or arrow cards. Ask the child to represent the number using structured resources. Ask: '*Which number is easy to add or take away from your number?'* Encourage them to use what they know about place value and number bonds and record matching number sentences.

- ✓ Develop patterns of connected calculations from known facts relating to all operations:
  - Facts for free: if I know 6 + 4 = 10, I also know 4 + 6 = 10, 10 – 4 = 6 and 10 – 6 = 4
  - Nearby facts: If I know 10 + 10 = 20, I also know 10 + 11 and 9 + 10 etc.
  - Place value: If I know 3 + 7 = 10, I also know 30 + 70 = 100, 300 + 700 = 100 0.3 + 0.7 = 1 etc.
  - Equivalence: If I know 15 + 5 = 20, I also know 14 + 6, 13 + 7, 12 + 8 etc.

Children also need support to use what they know about multiplication and division. Arrays provide a powerful image for making connections.

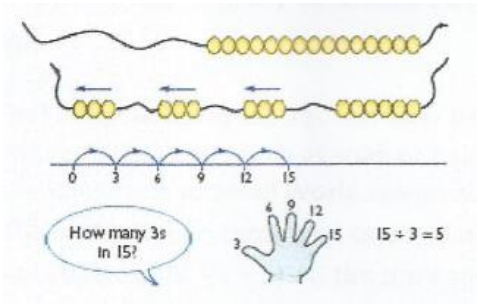




- ✓ Use a variety of counters or objects appropriate to the child. Set different calculations depending on the focus of the maths e.g. *'There are 20 cakes on a tray. Show me how many cakes 4 children would get each if they were shared equally. The children baked 15 cookies. How many packets could they sell if each packet contains 5 cookies?'*

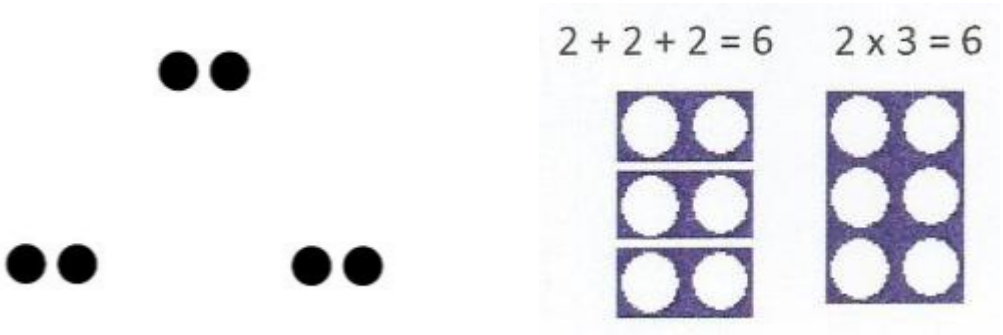
- ✓ Encourage the child to tell different number stories that match given calculations. Provide a model that can be adapted e.g. 4 x 10 = 40. *'There are 4 wheels on each car and there are ten cars so there are 40 wheels altogether. Now you make up a story about cartons of juice that come in packets of 4.'*

- ✓ Use bead strings and number lines to show the constant step size or jump



- ✓ Set a word problem to encourage the child to use practical apparatus and models and images to help work out and explain the problem. Include various structures for each operation e.g. grouping and sharing for division; repeated addition and scaling for multiplication.

- ✓ Using dot pattern or resources arranged in patterns. Ask: '*How many dots are there? Explain how you can work it out without counting all of the dots? What number sentences can record to match your methods?'*



- ✓ *'Here are 12 counters. Can you arrange them in equal rows? How many in each row? Record the matching number sentence. Can you arrange them in a different way?'*

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Appendix A

Maths Intervention Training Opportunities: Developed in Devon

In line with current expectations, that interventions will most commonly be focused on ensuring pupils are helped to keep up, the materials provided in each of these programmes can be used as part of immediate intervention sessions during teaching sequences or can also be used to run a more intensive intervention programme to support children who are working significantly below age-related expectations.

"From Counting to Calculating" is a comprehensive training programme which explores in depth how to support understanding and address misconceptions within early number and additive reasoning. It includes a set of flexible intervention materials, containing detailed activities, written to support children with developing conceptual understanding of number, addition and subtraction. The materials can be used as part of immediate intervention sessions during teaching sequences in Y1 and Y2 or as an intervention programme for children who are working significantly below age-related expectations in Year 2, KS2, KS3 and beyond. They cover key ideas in the Early Years Foundation Stage and in the National Curriculum for Y1 and Y2.

"Making a difference: developing additive reasoning" is a comprehensive training programme which explores in depth how to support understanding and address misconceptions within additive reasoning. It includes a set of flexible intervention materials, containing detailed activities, written to support children with developing conceptual understanding of addition and subtraction. The materials can be used as part of immediate intervention sessions during teaching sequences in Y2 to Y5 or as an intervention programme for children who are working significantly below age-related expectations in KS2, KS3 and beyond. They build on the understanding developed in KS1 (explored within From Counting to Calculating) and cover the expectations in the National Curriculum from Y2 up to the end of Y4.

"Fair and Equal: developing multiplicative reasoning" is a comprehensive training programme which explores in depth how to support understanding of and address misconceptions and difficult areas within multiplicative reasoning. The package includes a flexible set of intervention materials, containing detailed activities, written to support children with developing conceptual understanding and covering key ideas in multiplication, division and fractions in the National Curriculum from Y2 to Y6. The materials can be used to run an intensive programme to support children who are working significantly below age-related expectations in KS2, KS3 and beyond or can use the materials to deliver immediate follow-up sessions during teaching sequences in KS2.

To find out more, please contact Andy Tynemouth andy.tynemouth@babcockinternational.com

All training can also be run for clusters of schools; for details of costings please contact Dr Ruth Trundley ruth.trundley@babcockinternational.com